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MATHEMATICS
TARGET IIT JEE

BASIC MATHEMATICS & LOGARITHM

THEORY AND EXERCISE BOOKLET

CONTENTS

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JEE Syllabus :

Logarithms and their properties

A. NUMBER SYSTEM

(i) Natural Numbers : The counting numbers 1, 2, 3, 4,..... are called Natural Numbers. The set of natural numbers is denoted by N. Thus N = {1, 2, 3, 4,......}. N is also denoted by I^+ or Z^+

(ii) Whole Numbers : Natural numbers including zero are called whole numbers. The set of whole numbers, is denoted by W. Thus W = {0, 1, 2,......}. W is also called as set of non-negative integers.

(iii) Integers : The numbers..... – 3, – 2, –1, 0, 1, 2, 3..... are called integers and the set is denoted by I or Z. Thus I (or Z) = {...–3, –2, –1, 0, 1, 2, 3........}

(a) Set of positive integers, denoted by I^+ and consists of {1, 2, 3, ...}

(b) Set of negative integers, denoted by I^- and consists of {.........., –3, –2, –1}

(c) Set of non-negative integers {0, 1, 2, 3,..........}

(d) Set of non-positive integers {...., –3, –2, –1, 0}

(iv) Even Integers : Integers which are divisible by 2 are called even integers. e.g. 0, ± 2, ± 4,.....

(v) Odd Integers : Integers which are not divisible by 2 are called as odd integers. e.g. ± 1, ± 3, ...

(vi) Prime Number : Let 'p' be a natural number, 'p' is said to be prime if it has exactly two distinct factors, namely 1 and itself. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,......

Remark : (a) '1' is neither prime nor composite.

(b) '2' is the only even prime number.

(vii) Composite Number : Let 'a' be a natural number, 'a' is said to be composite if, it has atleast three distinct factors.

(viii) Co-prime Numbers : Two natural numbers (not necessarily prime) are coprime, if their H.C.F. (Highest common factor) is one. e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) etc.
These numbers are also called as **relatively prime** numbers.

Remark : (a) Number which are not prime are composite numbers (except 1)

(b) '4' is the smallest composite number.

(c) Two distinct prime numbers are always co-prime but converse need not be true.

(d) Consecutive numbers are always co-prime numbers.

(ix) Twin Prime Numbers : If the difference between two prime numbers is two, then the numbers are called as twin prime numbers. e.g. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

(x) Rational Numbers : All the numbers those can be represented in the form p/q, where p and q are integers and $q \neq 0$, are called rational numbers and their set is denoted by Q.

Thus Q = $\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\}$. It may be noted that every integer is a rational numbers. If not integer then either finite or recurring.

(xi) Irrational Numbers : There are real numbers which cannot be expressed in p/q form. These numbers are called irrational numbers and their set is denoted by Q^c or Q'. (i.e. complementary set of Q) e.g. $\sqrt{2}$, $1 + \sqrt{3}$, e, π etc. Irrational numbers can not be expressed as recurring decimals.

Remark : $e \approx 2.71$ is called Napier's constant and $\pi \approx 3.14$.

Surds : If a is not a perfect nth power, then $\sqrt[n]{a}$ is called a surd of the nth order.

In an expression of the form $\frac{a}{\sqrt{b}+\sqrt{c}}$, the denominator can be rationalized by multiplying numerator and the denominator by $\sqrt{b}-\sqrt{c}$ which is called the conjugate of $\sqrt{b}+\sqrt{c}$.

If x + $\sqrt{y}$ = a + $\sqrt{b}$ where x, y, a, b are rationals, then x = a and y = b.

Ex.1 Prove that $\log_3 5$ is irrational.

Sol. Let $\log_3 5$ is rational.

$\therefore$ $\log_3 5 = \frac{p}{q}$; where p and q are co-prime numbers

$\Rightarrow 3^{p/q} = 5 \Rightarrow 3^p = 5^q$. which is not possible, hence our assumption is wrong and $\log_3 5$ is irrational.

Ex.2 Simply (make the denominator rational) $\frac{12}{3+\sqrt{5}-2\sqrt{2}}$

Sol. The expression = $\frac{12(3+\sqrt{5}+2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2} = \frac{12(3+\sqrt{5}+2\sqrt{2})}{6+6\sqrt{5}}$

$$= \frac{2(3+\sqrt{5}+2\sqrt{2})(\sqrt{5}-1)}{(\sqrt{5}+1)\times(\sqrt{5}-1)} = \frac{2(2+2\sqrt{5}+2\sqrt{10}-2\sqrt{2})}{4} = 1+\sqrt{5}+\sqrt{10}-\sqrt{2}$$

Ex.3 Find the factor which will rationalize $\sqrt{3}+\sqrt[3]{5}$

Sol. Let x = $3^{1/2}$ and y = $5^{1/3}$. The L.C.M. of the denominators of the indices 2 and 3 is 6. Hence x^6 and y^6 are rational. Now $x^6 + y^6 = (x + y)(x^5 - x^4 y + x^3 y^2 - x^2y^3 + xy^4 - y^5)$
Hence the rationalizing factor required = $x^5 - x^4 y + x^3 y^2 - x^2 y^3 + xy^4 - y^5$ where x = $3^{1/2}$ and y = $5^{1/3}$.

Ex.4 Find the square root of 7 + 2$\sqrt{10}$

Sol. Let $\sqrt{7+2\sqrt{10}} = \sqrt{x}+\sqrt{y}$. Squaring, x + y + $2\sqrt{xy} = 7+2\sqrt{10}$

Hence x + y = 7 and xy = 10. These two relation give x = 5, y = 2. Hence $\sqrt{7+2\sqrt{10}} = \sqrt{5}+\sqrt{2}$

Remark : $\sqrt{\quad}$ symbol stands for the positive square root only.

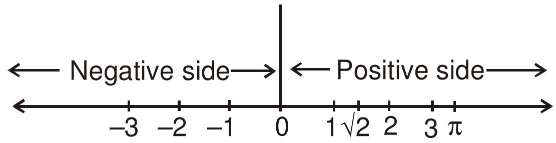
Ex.5 Prove that $\sqrt[3]{2}$ cannot be represented in the form p + $\sqrt{q}$, where p and q are rational (q > 0 and is not a perfect square).

Sol. Put $\sqrt[3]{2}$ = p + $\sqrt{q}$. Hence 2 = $p^3 + 3pq + (3p^2 + q)\sqrt{q}$,

Since q is not a perfect square, it must be $3p^2 + q = 0$, which is impossible.

(xii) Real Numbers : The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus $R = Q \cup Q^c$. Real numbers can be represented as points of a line. This line is called as real line or number line

All the real numbers follow the order property i.e. if there are two distinct real numbers a and b then either $a < b$ or $a > b$.



Remark :

(a) Integers are rational numbers, but converse need not be true.

(b) Negative of an irrational number is an irrational number.

(c) Sum of a rational number and an irrational number is always an irrational number e.g. $2 + \sqrt{3}$

(d) The product of a non zero rational number & an irrational number will always be an irrational number.

(e) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if $a = 0$.

(f) Sum, difference, product and quotient of two irrational numbers need not be an irrational number (it may be a rational number also).

(xiii) Complex Number : A number of the form $a + ib$ is called complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by C.

Remark : It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

Ex.6 Every number is one of the forms $5n$, $5n \pm 1$, $5n \pm 2$.

Sol. For if any number is divided by 5, the remainder is one of the numbers 0, 1, 2, 5 – 2, 5 – 1.

Ex.7 Every square number is one of the forms $5n$, $5n \pm 1$.

Sol. The square of every number is one of the forms $(5m)^2$, $(5m \pm 1)^2$, $(5m \pm 2)^2$. If those are divided by 5, the remainders are 0, 1, 4; and, since $4 = 5 - 1$, the forms are $5n$, $5n + 1$, and $5n - 1$.

Ex.8 Show that the number of primes in N is infinite.

Sol. Suppose the number of primes in N is finite. Let $\{p_1, p_2, \ldots, p_n\}$ be the set of primes in N such that $p_1 < p_2 < \ldots < p_n$. Consider $n = 1 + p_1p_2 \ldots p_n$. Clearly n is not divisible by any one of $p_1, p_2, \ldots, p_n$. Hence n itself is a prime and n has a prime divisor other than $p_1, p_2 \ldots p_n$. This contradicts that the set of primes is $\{p_1, p_2, \ldots, p_n\}$. Therefore the number of primes in N is infinite.

Ex.9 If x and y are prime numbers which satisfy $x^2 - 2y^2 = 1$, solve for x and y.

Sol. $x^2 - 2y^2 = 1$ gives $x^2 = 2y^2 + 1$ and hence x must be an odd number. If $x = 2n + 1$, then $x^2 = (2n + 1)^2 = 4n^2 + 4n + 1 = 2y^2 + 1$. Therefore $y^2 = 2n(n + 1)$. This means that y^2 is even and hence y is an even integer. Now, y is also a prime implies that $y = 2$. This gives $x = 3$. Thus the only solution is $x = 3$, $y = 2$.

B. DIVISIBILITY TEST :

(i) A number will be divisible by 2 iff the digit at the unit place is divisible by 2.

(ii) A number will be divisible by 3 iff the sum of its digits of the number is divisible by 3.

(iii) A number will be divisible by 4 iff last two digits of the number together are divisible by 4.

(iv) A number will be divisible by 5 iff digit at the unit place is either 0 or 5.

(v) A number will be divisible by 6 iff the digit at the unit place of the number is divisible by 2 & sum of all digits of the number is divisible by 3.

(vi) A number will be divisible by 8 iff the last 3 digits, all together, is divisible by 8.

(vii) A number will be divisible by 9 iff sum of all it's digits is divisible by 9.

(viii) A number will be divisible by 10 iff it's last digit is 0.

(ix) A number will be divisible by 11 iff the difference between the sum of the digits at even places and sum of the digits at odd places is a multiple of 11.
e.g. 1298, 1221, 123321, 12344321, 1234554321, 123456654321, 795432

Ex.10 Prove that :

(a) the sum $\overline{ab} + \overline{ba}$ is multiple of 11;

(b) a three-digit number written by one and the same digit is entirely divisible by 37.

Sol. **(a)** $\overline{ab} + \overline{ba}$ = (10a + b) + (10b + a) = 11(a + b);

(b) $\overline{aaa}$ = 100a + 10a + a = 111a = 37.3a.

Ex.11 Prove that the difference $10^{25} - 7$ is divisible by 3.

Sol. Write the given difference in the form $10^{25} - 7 = (10^{25} - 1) - 6$. The number $10^{25} - 1 = \underbrace{\overline{99..9}}_{25\text{ digits}}$ is divisible by 3(and 9). Since the numbers $(10^{25} - 1)$ and 6 are divisible by 3, the number $10^{25} - 7$, being their difference, is also divisible by 3 without a remainder.

Ex.12 If the number A 3 6 4 0 5 4 8 9 8 1 2 7 0 6 4 4 B is divisible by 99 then the ordered pair of digits (A, B) is

Sol. S_O = A + 37 ; S_E = B + 34 $\Rightarrow$ A – B + 3 = 0 or 11 and A + B + 71is a multiple of 9
$\Rightarrow$ A – B = –3 or 8 and A + B = 1 or 10 Ans. : (9, 1)

Ex.13 Consider a number N = 2 1 P 5 3 Q 4. Find the number of ordered pairs (P, Q) so that the number 'N is divisible by 44, is

Sol. S_O = P + 9, S_E = Q + 6 $\Rightarrow$ $S_O - S_E$ = P – Q + 3
'N' is divisible is 11 if P – Q +3 = 0, 11
P – Q = –3 ....(i) or P – Q = 8 ....(ii)
N is divisible by 4 if Q = 0, 2, 4, 6, 8
From Equation (i)
Q = 0 P = –3 (not possible) Q = 2P = –1 (not possible)

Q = 4 P = 1 Q = 6P = 3 Q = 8P = 5

$\therefore$ number of ordered pairs is 3

From equation (ii)

Q = 0 P = 8 Q = 2P = 10 (not possible) similarly $Q \neq 4, 6, 8$

$\therefore$ No. of ordered pairs is 1

$\therefore$ total number of ordered pairs, so that number 'N' is divisible by 44, is 4

Ex.14 Prove that the square of any prime number $p \geq 5$, when divided by 12, gives 1 as remainder.

Sol. When divided by 6, a natural number can give as a remainder only the numbers 0, 1, 2, 3, 4 and 5. Therefore, any natural number has one of the following forms :

6k, 6k + 1, 6k + 2, 6k + 3, 6k + 4, 6k + 5.

it is obvious that the numbers 6k, 6k + 2, 6k + 3, and 6k + 4 are composite. Therefore, the prime number $p \geq 5$ has the form 6k + 1 or 6k + 5.

If p = 6k + 1, then $p^2 = (6k + 1)^2 = 36k^2 + 12k + 1$.

If p = 6k + 5, then $p^2 = (6k + 5)^2 = 36k^2 + 60k + 25 = 12(3k^2 + 5k + 2) + 1$.

Thus, in both cases, when dividing p^2 by 12, the remainder is equal to 1.

Ex.15 Prove that for every positive integer n, $1^n + 8^n - 3^n - 6^n$ is divisible by 10.

Sol. Since 10 is the product of two primes 2 and 5, it will suffice to show that the given expression is divisible both by 2 and 5. To do so, we shall use the simple fact that if a and b be any positive integers, then $a^n - b^n$ is always divisible by a –b.

Writing $A \equiv 1^n + 8^n - 3^n - 6^n$, $= (8^n - 3^n) - (6^n - 1^n)$,

we find that $8^n - 3^n$ and $6^n - 1^n$ are both divisible by 5, and consequently A is divisible by 5 (= 8 – 3 = 6 – 1). Again, writing $A = (8^n - 6^n) - (3^n - 1^n)$, we find that A is divisible by 2(= 8 – 6 = 3 – 1). Hence A is divisible by 10.

C. (1) LCM AND HCF

(i) HCF is highest common factor between any two or more numbers (or algebraic expression) when only take numbers Its called highest common divisor.

(ii) LCM is least common multiple between any two or more numbers (or algebraic expression)

(iii) Multiplication of LCM and HCF of two numbers is equal to multiplication of two numbers.

(iv) LCM of $\left(\frac{a}{b}, \frac{p}{q}, \frac{\ell}{m}\right) = \frac{\text{LCM of } (a, p, \ell)}{\text{HCF of } (b, q, m)}$

(v) HCF of $\left(\frac{a}{b}, \frac{p}{q}, \frac{\ell}{m}\right) = \frac{\text{HCF of } (a, p, \ell)}{\text{LCM of } (b, q, m)}$

(vi) LCM of rational and irrational number is not defined.

(2) Remainder Theorem : Let P(x) be any polynomial of degree greater than or equal to one and 'a' be any real number. If P(x)is divided (x – a), then the remainder is equal to P(a).

(3) Factor Theorem : Let P(x) be polynomial of degree greater than of equal to 1 and 'a' be a real number such that P(a) = 0, then (x – a) is a factor of P(x). Conversely, if (x – a) is a factor of P(x), then P(a) = 0.

(4) Some Important Identities :

(i) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$

(ii) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(v) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(vi) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$

(vii) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$

(viii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(ix) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(x) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

(xi) $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$

(xii) $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

Remark : (1) $ab + bc + ca = abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(2) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(5) Definition Of Indices : If 'a' any none zero real or imaginary number and m is positive integer than $a^m = a.a.a. ....a$ (m times) where 'a' is base 'm' is indices

Law of Indices :

(i) $a^0 = 1, \ (a \neq 0)$ **(ii)** $a^{-m} = \frac{1}{a^m}, \ (a \neq 0)$

(iii) $a^{m+n} = a^m . a^n$, where m and n real numbers

(iv) $a^{m-n} = \frac{a^m}{a^n}$, where m and n real numbers, $a \neq 0$

(v) $(a^m)^n = a^{mn}$ **(vi)** $a^{p/q} = \sqrt[q]{a^p}$

Ex.16 Find p and q so that (x + 2) and (x – 1) may be factors of the polynomial $f(x) = x^3 + 10x^2 + px + q$.

Sol. Since (x + 2) is a factor f(–2) must be zero $\therefore -8 + 40 - 2p + q = 0$ ...(1)

Since (x – 1) is a factor, f(1) must be zero $\therefore 1 + 10 + p + q = 0$ ...(2)

From (1) and (2), by solving we get p = 7 and q = –18

Ex.17 Show that (2x + 1) is a factor of the expression $f(x) = 32x^5 - 16x^4 + 8x^3 + 4x + 5$.

Sol. Since (2x + 1) is to be a factor of f(x), $f\left(-\frac{1}{2}\right)$ should be zero.

$f\left(-\frac{1}{2}\right) = 32\left(-\frac{1}{2}\right)^5 - 16\left(-\frac{1}{2}\right)^4 + 8\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right) + 5$. Hence (2x + 1) is a factor of f(x).

Ex.18 Without using the Remainder theorem, find the remainder when
$f(x) = x^6 - 19x^5 + 69x^4 - 151x^3 + 229x^2 + 166x + 26$ is divided by x – 15.

Sol. f(x) can be written as
$(x^6 - 15x^5) - 4(x^5 - 15x^4) + 9(x^4 - 15x^3) - 16(x^3 - 15x^2) - 11(x^2 - 15x) + (x - 15) + 41$
or as $f(x) = x^5 (x - 15) - 4x^4(x - 15) + 9x^3(x - 15) - 16x^2(x - 15) - 11x(x - 15) + (x - 15) + 41$
Since the first six terms have x – 15 as a factor, remainder = 41.

Ex.19 Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Sol. Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ and $g(x) = x^2 - 3x + 2$ be the given polynomials.
Then $g(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$
In order to prove that f(x) is exactly divisible by g(x), it is sufficient to prove that x – 1 and x – 2 are factors of f(x). For this it is sufficient to prove that f(1) = 0 and f(2) = 0.
Now, $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$
$\Rightarrow$ $f(1) = 2 \times 1^4 - 6 \times 1^3 + 3 \times 1^2 + 3 \times 1 - 2$ and, $f(2) = 2 \times 2^2 - 6 \times 2^3 \times 2^2 + 3 \times 2 - 2$
$\Rightarrow$ f(1) = 2 – 6 + 3 + 3 – 2 and f(2) = 32 – 48 + 12 + 6 – 2
$\Rightarrow$ f(1) = 8 – 8 and f(2) = 50 – 50 $\Rightarrow$ f(1) = 0 and f(2) = 0
$\Rightarrow$ (x – 1) and (x – 2) are factors of f(x) $\Rightarrow$ g(x) = (x – 1) (x – 2) is a factors of f(x).
Hence, f(x) is exactly divisible by g(x).

Ex.20 Using factor theorem, show that a – b, b – c and c – a are the factors of
$a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Sol. By factor theorem, a – b will be a factor of the given expression if it vanishes by substituting a = b in it.
substituting a = b in the given expression, we have $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$
$= b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2) = b^3 - bc^2 + bc^2 - b^3 + c(b^2 - b^2) = 0$
$\therefore$ (a – b) is a factor of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.
Similarly, we can show that (b – c) and (c – a) are also factors of the given expression.
Hence, (a – b), (b – c) and (c – a) are factors of the given expression.

Ex.21 Show that x – 2y is a factor or $3x^3 - 2x^2 y - 13xy^2 + 10y^3$.

Sol. Let $f(x) = 3x^3 - 2x^2y - 13xy^2 + 10y^3$
Then $f(2y) = 3(2y)^3 - 2y(2y)^2 - 13y^2(2y) + 10y^3 = 24y^3 - 8y^3 - 26y^3 + 10y^3 = 0$
Hence x – 2y is a factor of f(x).

Ex.22 Show that $a^n - b^n$ is divisible by a – b if n is any positive integer odd or even.

Sol. Let $a^n - b^n = f(a)$. By Remainder theorem, $f(b) = b^n - b^n = 0$ (replacing a by b)
$\therefore$ a – b is a factor of $a^n - b^n$.

Ex.23 Show that $a^n - b^n$ is divisible by (a + b) when n is an even positive integer. but not if n is odd.

Sol. Let $a^n - b^n = f(a)$. Now $f(-b) = (-b)^n - b^n = b^n - b^n = 0$ if n is even and hence a + b is a factor of $a^n - b^n$
If n is odd, $f(-b) = -b^n - b^n = -2b^n \neq 0$.

Ex.24 If a + b + c = 0, prove that $a^4 + b^4 + c^4 = 2(b^2 c^2 + c^2 a^2 + a^2 b^2) = 1/2(a^2 + b^2 + c^2)^2$

Sol. Squaring both sides of the relation $(a^2 + b^2 + c^2)^2 = [-2(bc + ca + ab)]^2$
$= 4\{b^2 c^2 + c^2 a^2 + a^2 b^2 + 2\{bc.\ ca + ca.\ ab + ab.\ bc\}$,
$= 4(b^2 c^2 + c^2 a^2 + a^2 b^2) + 8abc(a + b + c) = 4(b^2 c^2 + c^2 a^2 + a^2 b^2)$, since a + b + c = 0.
Therefore, $2(b^2c^2 + c^2 a^2 + a^2b^2) = 1/2\ (a^2 + b^2 + c^2)^2$.
Also $(a^2 + b^2 + c^2)^2 = (a^4 + b^4 + c^4) + 2(b^2c^2 + c^2 a^2 + a^2b^2)$,
so that $4(b^2c^2 + c^2 a^2 + a^2b^2) = (a^4 + b^4 + c^4) + 2(b^2c^2 + c^2 a^2 + a^2b^2)$
whence $a^4 + b^4 + c^4 = 2(b^2c^2 + c^2 a^2 + a^2b^2)$.

Ex.25 Factor the following expression $(x + y + z)^3 - x^3 - y^3 - z^3$.

Sol. We have $(x + y + z)^3 = x^3 + y^3 + z^3 + 3x^2 (y + z) + (3y^2 (x + z) + 3z^2 (x + y) + 6xyz$.
Hence $(x + y + z)^3 - x^3 - y^3 - z^3 = 3\{x^2y + x^2z + y^2x + y^2z + z^2x + z^2y + 2xyz\}$
$= 3\{z(x^2 + y^2 + 2xy) + z^2(x + y) + xy(x + y)\} = 3(x + y)\{z(x + y) + z^2 + xy\}$
$= 3(x + y)(x + z)(y + z)$.
Thus, $(x + y + z)^3 - x^3 - y^3 - z^3 = 3(x + y)(x + z)(y + z)$.

Ex.26 Prove that if a + b + c = 0, then $(a^2 + b^2 + c^2)^2 = 2(a^4 + b^4 + c^4)$.

Sol. We have $(a + b + c)^2 = 0 \Rightarrow a^2 + b^2 + c^2 = -2(ab + ac + bc)$.
Squaring both members of the latter equality, we get
$(a^2 + b^2 + c^2)^2 = 4[a^2b^2 + a^2c^2 + b^2c^2 + 2a^2bc + 2b^2ac + 2c^2ab]$
$= 4[a^2b^2 + a^2c^2 + b^2c^2 + 2abc(a + b + c)] = 4[a^2b^2 + a^2c^2 + b^2c^2]$ ....(1)
On the other hand,
$(a^2 + b^2 + c^2)^2 = (a^4 + b^4 + c^4) + 2(a^2b^2 + a^2c^2 + b^2c^2)$.
$4(a^2b^2 + a^2c^2 + b^2c^2) + 2(a^4 + b^4 + c^4) = 2(a^2 + b^2 + c^2)^2$ ....(2)
by (1) and (2) we get the required result $\Rightarrow (a^2 + b^2 + c^2)^2 = 2(a^4 + b^4 + c^4)$

Ex.27 Show that from the equality $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$ follows $\frac{1}{a^n}+\frac{1}{b^n}+\frac{1}{c^n}=\frac{1}{a^n+b^n+c^n}$ in n is odd.

Sol. Reducing the original equality to a common denominator and cancelling it out,
we get (after some transformations) (a + b) (a + c) (b + c) = 0 ...(1)
But the second equality (which is to be proved) can also be reduced to the form
$(a^n + b^n)(a^n + c^n)(b^n + c^n) = 0$. ...(2)
It is quite obvious, that with an odd n equality (2) follows from (1), since if, for instance, a + b = 0, then a = –b and $a^n + b^n = a^n + (-a)^n = a^n - a^n = 0$.

Ex.28 Prove that $1 + b_2 + b_2b_3 + ... + b_2b_3 ...... b_n = \cfrac{1}{1-\cfrac{b_2}{b_2+1-\cfrac{b_3}{b_3+1-\ddots-\cfrac{b_n}{b_n+1}}}}$.

Sol. Let us denote the continued fraction on the right by $\frac{P_n}{Q_n}$. We have to prove that

$$\frac{P_n}{Q_n} = 1 + b_2 + b_2b_3 + ... + b_2b_3 ...... b_n.$$

We have $\frac{P_1}{Q_1} = \frac{1}{1}, \frac{P_2}{Q_2} = \frac{b_2+1}{1}$.

Therefore we may take $P_1 = 1$, $Q_1 = 1$, $P_2 = b_2 + 1$, $Q_2 = 1$. Then, using the method of induction, it is easy to prove that $P_n = 1 + b_2 + b_2b_3 + ... + b_2b_3 ...... b_n$, $Q_n = 1$,
and, consequently, out equality is also true.

Ex.29 Solve the equation, $\frac{x-ab}{a+b} + \frac{x-bc}{b+c} + \frac{x-ca}{1+a} = a + b + c$. What happens if $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = 0$

Sol. $\left(\frac{x-ab}{a+b} - c\right) + \left(\frac{x-bc}{b+c} - a\right) + \left(\frac{x-ca}{c+a} - b\right) = 0$

$\Rightarrow (x - (ab + bc + ca))\left[\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}\right] = 0 \Rightarrow x = ab + bc + ca.$

If $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = 0 \Rightarrow$ the given equation becomes an identity & is true for all $x \in R$

D. (1) RATIO

(i) If A and B be two quantities of the same kind, then their ratio is A : B; which may be denoted by the fraction $\frac{A}{B}$ (This may be an integer or fraction)

(ii) A ratio may represented in a number of ways e.g. $\frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb} = ...$ where m, n,...... are non-zero numbers.

(iii) To compare two or more ratio, reduce them to common denominator.

(iv) Ratio between two ratios may be represented as the ratio of two integers e.g. $\frac{a}{b} : \frac{c}{d} : \frac{a/b}{c/d} = \frac{ad}{bc}$ or ad : bc.

(v) Ratios are compounded by multiplying them together i.e. $\frac{a}{b} . \frac{c}{d} . \frac{e}{f} ...... = \frac{ace}{bdf} ....$

(vi) If a : b is any ratio then its duplicate ratio is $a^2 : b^2$; triplicate ratio is $a^3 : b^3$..... etc.

(vii) If a : b is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$; sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

(2) PROPORTION

When two ratios are equal, then the four quantities compositing them are said to be proportional.

If $\frac{a}{b} = \frac{c}{d}$, then it is written as a : b = c : d or a : b : : c : d

(i) 'a' and 'd' are known as extremes and 'b and c' are known as means.

(ii) An important property of proportion Product of extremes = product of means.

(iii) If a : b = c : d, then b : a = d : c (Invertando)

(iv) If a : b = c : d, then a : c = b : d (Alternando)

(v) If a : b = c : d, then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

(vi) If a : b = c : d, then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

(vii) If a : b = c : d, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo and dividendo)

(viii) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $\frac{a+c+e+.....}{b+d+f+......} = \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

(ix) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $= \frac{xa+yc+ze+......}{xb+yd+zf+......}$

(x) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $= \left(\frac{xa^n+yc^n+ze^n}{xb^n+yd^n+zf^n}\right)^{1/n}$

Ex.30 If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$, then find x : y : z.

Sol. Each $= \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}} = \frac{2(x+y+z)}{9} = \frac{x+y+z}{9/2}$

and therefore each $= \frac{(x+y+z)-(y+z)}{\frac{9}{2}-3} = \frac{(x+y+z)-(x+z)}{\frac{9}{2}-4} = \frac{(x+y+z)-(x+y)}{\frac{9}{2}-2}$

$= \frac{x}{3/2} = \frac{y}{1/2} = \frac{z}{5/2} \Rightarrow$ x : y : z = 3 : 1 : 5

Ex.31 If a(y + z) = b(z + x) = c(x + y), then show that $\frac{a-b}{x^2-y^2} = \frac{b-c}{y^2-z^2} = \frac{c-a}{z^2-x^2}$

Sol. Given condition can be written as $\frac{y+z}{1/a}=\frac{z+x}{1/b}=\frac{x+y}{1/c}=k$ ....(1)

$$\text{Each}=\frac{(z+x)-(y+z)}{\frac{1}{b}-\frac{1}{a}}=\frac{(x+y)-(x+z)}{\frac{1}{c}-\frac{1}{b}}=\frac{(y+z)-(x+y)}{\frac{1}{a}-\frac{1}{c}}=\frac{x-y}{\frac{a-b}{ab}}=\frac{y-z}{\frac{b-c}{bc}}=\frac{z-x}{\frac{c-a}{ca}}=k \quad ....(2)$$

Form (1) and (2), we get by multiplying

$$\frac{x^2-y^2}{\frac{a-b}{abc}}=\frac{y^2-z^2}{\frac{b-c}{abc}}=\frac{z^2-x^2}{\frac{c-a}{abc}}\Rightarrow\frac{x^2-y^2}{a-b}=\frac{y^2-z^2}{b-c}=\frac{z^2-x^2}{c-a}\Rightarrow\frac{a-b}{x^2-y^2}=\frac{b-c}{y^2-z^2}=\frac{c-a}{z^2-x^2}$$

Ex.32 If $x=\frac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$, show that $3bx^2 - 4ax + 3b = 0$.

Sol. Taking the left hand side as $\frac{x}{1}$, using componendo and dividendo, $\frac{x+1}{x-1}=\frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}$

Squaring, $\frac{(x+1)^2}{(x-1)^2}=\frac{2a+3b}{2a-3b}$ and again applying componendo and dividendo $\frac{x^2+1}{2x}=\frac{2a}{3b}$ which gives the answer on cross multiplication.

Ex.33 If $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$, then show that $\frac{9x}{2b+2c-a}=\frac{9y}{2c+2a-b}=\frac{9z}{2a+2b-c}$

Sol. Since $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$, each is equal to

$\frac{2(2z+2x-y)+2(2x+2y-z)-(2y+2z-x)}{2b+2c-a}$ by a theorem quoted earlier

$=\frac{9x}{2b+2c-a}$ on simplification.

Similarly, each $=\frac{9y}{2c+2a-b}$ and $\frac{9z}{2a+2b-c}$ and hence the result.

Ex.34 Solve : $\frac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}}=2$

Sol. Writing the R.H.S. as $\frac{2}{1}$ and using componendo and dividendo,

$$\frac{(\sqrt{2+x}+\sqrt{2-x})+(\sqrt{2+x}-\sqrt{2-x})}{(\sqrt{2+x}+\sqrt{2-x})-(\sqrt{2+x}-\sqrt{2-x})}=\frac{2+1}{2-1} \qquad \text{(i.e.)}\ \frac{\sqrt{2+x}}{\sqrt{2-x}}=\frac{3}{1}$$

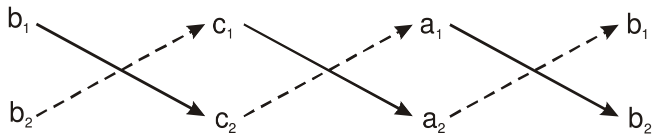
Squaring, $\frac{2+x}{2-x}=\frac{9}{1}$ and again applying componendo and dividendo $\frac{4}{2x}=\frac{10}{8}$ and hence $x=\frac{8}{5}$

E. CROSS MULTIPLICATION RULE

If two equations containing three unknown are $a_1x + b_1y + c_1z = 0$ ...(i) $a_2x + b_2y + c_2z = 0$ ....(ii)

Then by the rule of cross multiplication $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - a_2c_1} = \dfrac{z}{a_1b_2 - a_2b_1}$ ...(iii)

In order to write down the denominators of x, y and z in (3) apply the following rule, "write down the coefficients of x, y and z in order beginning with the coefficients of y and repeat them as in the diagram".



Multiply the coefficients across in the way indicated by the arrows; remembering that informing the products any one obtained by descending is positive and any one obtained by ascending is negative.

Ex.35 Solve $\left.\begin{array}{l} 2x + 3y - 8 = 0 \\ 3x - 4y + 5 = 0 \end{array}\right\}$ by rule of cross multiplication.

Sol. Note : (i) Write all the terms of L.H.S. with R.H.S = 0 (ii) Negative sign is part of the coefficient.

$$\begin{array}{cccc} 3 & (-8) & 2 & 3 \\ (-4) & 5 & 3 & (-4) \end{array} \qquad \frac{x}{(3)(5)-(-4)(-8)} = \frac{y}{(-8)(3)-(5)(2)} = \frac{1}{2(-4)-(3)(3)}$$

(i.e.) $\dfrac{x}{-17} = \dfrac{y}{-34} = \dfrac{1}{-17}$ or $\dfrac{x}{1} = \dfrac{y}{2} = 1$ and hence x = 1, y = 2.

Ex.36 Solve : $2x - 3y + 4z = 0$; $7x + 2y - 6z = 0$; $4x + 3y + z = 37$

Sol. From the first two equations we have

$$\begin{array}{cccc} (-3) & 4 & 2 & -3 \\ 2 & (-6) & 7 & 2 \end{array} \qquad \frac{x}{10} = \frac{y}{40} = \frac{z}{25} \text{ or } \frac{x}{2} = \frac{y}{8} = \frac{z}{5} = k \text{ (say)}$$

Then x = 2k, y = 8k, z = 5k

Substituting these values of x, y, z in the third equation, k (8 + 24 + 5) = 37 $\Rightarrow$ k = 1

Hence x = 2, y = 8, z = 5

F. INTERVALS

Intervals are subsets of R and generally its used to find domain or inequality. If a and b are two real numbers such that a < b then we can defined for types of intervals

Open Interval	(a, b)	$\{x : a < x < b\}$ i.e. extreme points are not includes
Closed Interval	[a, b]	$\{x : a \le x \le b\}$ i.e. extreme points are includes It can possible when a and b are finite
Semi-Open Interval	(a, b]	$\{x : a < x \le b\}$ i.e. a is not include and b is include
Semi-Closed Interval	[a, b)	$\{x : a \le x < b\}$ i.e. a is include and b is not include

G. LOGARITHM OF A NUMBER

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N.
This number is designated as $\log_a N$.
Hence $\ell og_a N = x \Leftrightarrow a^x = N$, $a > 0$, $a \neq 1$ & $N > 0$
If $a = 10$, then we write log b rather than $\ell og_{10} b$
$a = e$, we write $\ell n\, b$ rather than $\ell og_e b$
The existence and uniqueness of the number $\ell og_a N$ follows from the properties of an exponential functions.
From the definition of the logarithm of the number N to the base 'a', we have an identity :

$$a^{\log_a N} = N,\ a > 0,\ a \neq 1\ \&\ N > 0$$

This is known as the **FUNDAMENTAL LOGARITHMIC IDENTITY.**

Remark : $\ell og_a 1 = 0$ $(a > 0, a \neq 1)$
$\ell og_a a = 1$ $(a > 0, a \neq 1)$
$\ell og_{1/a} a = -1$ $(a > 0, a \neq 1)$

Remember : $\ell og_{10} 2 = 0.3010$, $\ell og_{10} 3 = 0.4771$, $\ell n\, 2 = 0.693$, $\ell n\, 10 = 2.303$

The principal properties of logarithms :
Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ and α is any real number then ;
(i) $\ell og_a(M.N) = \ell og_a M + \ell og_a N$ **(ii)** $\ell og_a(M/N) = \ell og_a M - \ell og_a N$ **(iii)** $\ell og_a M^\alpha = \alpha.\ \ell og_a M$

(iv) $\ell og_{a^\beta} M = \dfrac{1}{\beta}\ \ell og_a M$ **(v)** $\ell og_b M = \dfrac{\ell og_a M}{\ell og_a b}$ (base change theorem)

Remark : $\ell og_b a . \ell og_a b = 1 \Leftrightarrow \ell og_b a = \dfrac{1}{\ell og_a b}$ $\ell og_b a.\ \ell og_c b.\ \ell og_a c = 1$

$\ell og_y x.\ \ell og_z y.\ \ell og_a z = \ell og_a x.$ $e^{\ln a^x} = a^x$

Logarithmic equations : $\ell og_a x = \ell og_a y$ possible iff $x = y$ i.e. $\ell og_a x = \ell og_a y \Leftrightarrow x = y$
Always check the validity of the given equation i.e. $x > 0$, $y > 0$, $a > 0$, $a \neq 1$

Common and natural logarithm : $\ell og_{10} N$ is referred as a common logarithm and $\ell og_e N$ is called as natural logarithm of N to the base Napierian and is popularly written as ℓn N. Note that e is an irrational quantity lying between 2.7 to 2.8 **Note that** $e^{\ell n\, x} = x$.

Characteristic & Mantissa : The common logarithm of a number consists of two parts, integral and fractional, of which the integral part may be zero or an integer (+ve or –ve) and the fractional part a decimal, less than one and always positive.
The integral part is called the characteristic and the decimal part is called the mantissa. It should be noted that, if the characteristic of the logarithm of N is p then number of significant digit in N = p + 1 if p is the non negative characteristic of log N. Number of zeros after decimal before a significant figure start is p – 1

Properties of monotonocity of logarithm :

(i) For a > 1 the inequality $0 < x < y$ & $\ell og_a x < \ell og_a y$ are equivalent
(ii) For $0 < a < 1$ the inequality $0 < x < y$ & $\ell og_a x > \ell og_a y$ are equivalent
(iii) If $a > 1$ then $\ell og_a x < p \Rightarrow 0 < x < a^p$
(iv) If $a > 1$ then $\ell og_a x > p \Rightarrow x > a^p$
(v) If $0 < a < 1$ then $\ell og_a x < p \Rightarrow x > a^p$
(vi) If $0 < a < 1$ then $\ell og_a x > p \Rightarrow 0 < x < a^p$

Note that :

(a) If the number & the base are on one side of the unity, then the logarithm is positive; If the number and the base are on different sides of unity, then the logarithm is negative.
(b) The base of the logarithm 'a' must not equal unity otherwise numbers not equal to unity will not have a logarithm & any number will be the logarithm of unity.
(c) For a non negative number 'a' & $n \ge 2$, $n \in N\ \sqrt[n]{a} = a^{1/n}$

Ex.37 Compute $\sqrt{\left(\frac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}}}$

Sol. Using in succession the laws of logarithms and exponents we compute the radicand:

$$\left(\frac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}} = \frac{1}{27}\cdot(\sqrt{27})^{\frac{1}{2}\log_9 13} = \frac{1}{27}\cdot(3^{\log_3 13})^{3/8} = 3^{-3}.\ 13^{3/8}$$

whence it is clear that the given number is equal to $3^{-3/2}.\ 13^{3/16}$.

Ex.38 Compute $\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{b}}$ if $\log_{ab} a = 4$.

Sol. By the laws of logarithms we have $\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{b}} = \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b = \frac{4}{3} - \frac{1}{2}\log_{ab} b$

It remains to find the quantity $\log_{ab} b$. Since $1 = \log_{ab} ab = \log_{ab} a + \log_{ab} b = 4 + \log_{ab} b$

It follows that $\log_{ab} b = -3$ and so $\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{b}} = \frac{4}{3} - \frac{1}{2}.(-3) = \frac{17}{6}$

Ex.39 Compute the value of $\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$.

Sol. $\frac{1}{\log_2 36} + \frac{1}{\log_3 36} = \log_{36}2 + \log_{36}3 = \log_{36}6 = \frac{1}{2}$

Ex.40 If $\log_{x-3}(2x - 3)$ is a meaningful quantity then find the interval in which x must lie.
Sol. $x - 3 > 0,\ x - 3 \neq 1$ and $2x - 3 > 0 \Rightarrow x > 3,\ x \neq 4$ and $x > 3/2 \Rightarrow (3, 4) \cup (4, \infty)$

Ex.41 For $x \ge 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x + 2)$?

Sol. $\log\frac{(x^3 - 4x^2 + x + 26)}{(x+2)} = \log\frac{(x^2 - 6x^2 + 13)(x+2)}{(x+2)} = \log(x^2 - 6x + 13)$ [$\because\ x \neq -2$]

$= \log\{(x - 3)^2 + 4\}$ $\therefore$ Minimum value is log 4 when x = 3

Ex.42 Given $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \dfrac{2}{s^3+1}$. Write $\log_2 \dfrac{a^2b^5}{c^4}$ as a function of 's' (a, b, c > 0, $c \neq 1$).

Sol. Given $\log_2 a = s$ ....(1) $\log_2 b = 2s^2$ ....(2) $\log_8 c^2 = \dfrac{s^3+1}{2}$ ....(3)

$\Rightarrow \dfrac{2\log c}{3\log 2} = \dfrac{s^3+1}{2} \Rightarrow 4\log_2 c = 3(s^3+1)$ ....(4)

to find $2\log_2 a + 5\log_2 b - 4\log_2 c \Rightarrow 2s + 10s^2 - 3(s^3+1)$

Ex.43 If log25 = *a* and log 225 = *b*, then find the value of $\log\left(\left(\dfrac{1}{9}\right)^2\right) + \log\left(\dfrac{1}{2250}\right)$ in terms of *a* and *b* (base of the log is 10 everywhere).

Sol. log 25 = a; log 225 = b

2 log 5 = a ; log(25 · 9) = b or log 25 + 2 log 3 = b $\Rightarrow$ 2 log 3 = b – a ....(1)

now $\log\left(\dfrac{1}{9}\right)^2 + \log\left(\dfrac{1}{2250}\right) = -2\log 9 - \log 2250 = -4\log 3 - [\log 225 + \log 10]$

$= -2(b-a) - [b+1] = -2b + 2a - b - 1 = 2a - 3b - 1$

Ex.44 Compute $\log_6 16$ if $\log_{12} 27 = a$

Sol. The chain of transformations $\log_6 16 = 4\log_6 2 = \dfrac{4}{\log_2 6} = \dfrac{4}{1+\log_2 3}$

shows us that we have to know $\log_2 3$ in order to find $\log_6 16$. We find it from the condition $\log_{12} 27 = a$:

$a = \log_{12} 27 = 3\log_{12} 3 = \dfrac{3}{\log_3 12} = \dfrac{3}{1+2\log_3 2} = \dfrac{3}{1+\dfrac{2}{\log_2 3}} = \dfrac{3\log_2 3}{2+\log_2 3}$

which means that $\log_2 3 = \dfrac{2a}{3-a}$ (note that, obviously, $a \neq 3$). We finally have $\log_6 16 = \dfrac{4(3-a)}{3+a}$.

Ex.45 If $\log_6 15 = \alpha$ and $\log_{12} 18 = \beta$ then compute the value of $\log_{25} 24$ in terms of α & β.

Sol. $\alpha = \dfrac{1+\log_3 5}{1+\log_3 2}$; $\beta = \dfrac{2+\log_3 2}{1+2\log_3 2}$

Let $\log_3 2 = x$ and $\log_3 5 = y$

$1 + y = \alpha(1+x)$ ....(1) $2 + x = \beta(2x+1)$ ....(2)

From (2) $x = \dfrac{2-\beta}{2\beta-1}$ ....(3) Putting this value of x in (1) $y = \dfrac{\alpha(1+\beta)-(2\beta-1)}{2\beta-1}$ ....(4)

Now $\log_{25} 24 = \dfrac{3x+1}{2y}$. Substitute the value of x and y to get $\log_{25} 24 = \dfrac{5-\beta}{2\alpha + 2\alpha\beta - 4\beta + 2}$

Ex.46 Suppose that *a* and *b* are positive real numbers such that $\log_{27}a + \log_9 b = \frac{7}{2}$ and $\log_{27}b + \log_9 a = \frac{2}{3}$. Find the value of the *ab*.

Sol. $\log_{27}a + \log_9 b = \frac{7}{2} \Rightarrow \frac{1}{3}\log_3 a + \frac{1}{2}\log_3 b = \frac{7}{2}$; $\log_{27}b + \log_9 a = \frac{2}{3} \Rightarrow \frac{1}{3}\log_3 b + \frac{1}{2}\log_3 a = \frac{2}{3}$

adding the equation $\frac{1}{3}\log_3(ab) + \frac{1}{2}\log_3(ab) = \frac{7}{2} + \frac{2}{3} = \frac{25}{6}$

$\frac{5}{6}\log_3(ab) = \frac{25}{6} \quad \Rightarrow \quad \log_3(ab) = 5 \quad \Rightarrow \quad ab = 3^5 = 243$

Ex.47 Let $x = (0.15)^{20}$. Find the characteristic and mantissa in the logarithm of x, to the base 10. Assume $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$.

Sol. $\log x = \log(0.15)^{20} = 20 \log\left(\frac{15}{100}\right)$

$= 20[\log 15 - 2] \quad = 20[\log 3 + \log 5 - 2] \quad = 20[\log 3 + 1 - \log 2 - 2]$

$= 20[-1 + \log 3 - \log 2] \quad = 20[-1 + 0.477 - 0.301] \quad = -20 \times 0.824 = -16.48 = \overline{17}.52$

hence characteristic = – 17 and mantissa = 0.52

Ex.48 Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7

Sol. $\log_7 N = x$ (where $3 \le x < 4$) $\Rightarrow 7^3 \le N < 7^4 \Rightarrow$ number of integers are 2058

Ex.49 How many digits are contained in the number 2^{75} ?

Sol. Computing $\log 2^{75}$, we have $\log 2^{75} = 75. \log 2 \approx 75. (0.3010) = 22.5750$.

Consequently, the characteristic of this common logarithm is equal to 22. Therefore, $2^{75} = a. 10^{22}$, where $1 \le a < 10$, a is an integer, and, hence the number 2^{75} has 23 digits.

Ex.50 If $\log_2(\log_2(\log_3 x)) = \log_2(\log_3(\log_2 y)) = 0$ then find the value of (x + y).

Sol. $\log_2(\log_2(\log_3 x)) = 0 \quad \Rightarrow \quad \log_2(\log_3 x) = 1 \quad \Rightarrow \quad \log_3 x = 2 \quad \Rightarrow \quad x = 9$

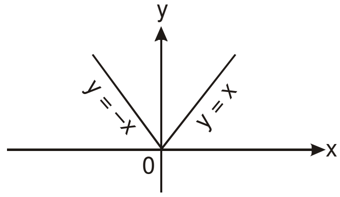
|||ly $\log_2(\log_3(\log_2 y)) = 0 \quad \Rightarrow \quad \log_3(\log_2 y) = 1 \quad \Rightarrow \quad \log_2 y = 3 \quad \Rightarrow \quad y = 8$

$\therefore \quad x + y = 17$

H. ABSOLUTE VALUE FUNCTION / MODULUS FUNCTION :

A function $y = |x|$ is called the absolute value function or

Modulus function. It is defined as : $y = |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$



EXERCISE – I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. If A & B are two rational numbers and AB, A + B and A – B are rational numbers, then A/B is
(A) always rational (B) never rational
(C) rational when $B \neq 0$ (D) rational when $A \neq 0$
Sol.

2. Every irrational number can be expressed on the number line. This statement is
(A) always true (B) never true
(C) true subject to some condition
(D) None of these
Sol.

3. The multiplication of a rational number 'x' and an irrational number 'y' is
(A) always rational (B) rational except when $y = \pi$
(C) always irrational (D) irrational except when x = 0
Sol.

4. If x, y are rational numbers such that
$(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$ then
(A) x = 1, y = 1 (B) x = 2, y = 1 (C) x = 5, y = 1
(D) x & y can take infinitely many values
Sol.

5. The number of real roots of the equation
$(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ is
(A) 0 (B) 1 (C) 2 (D) 3
Sol.

6. If a, b, c are real, then
a (a – b) + b (b – c) + c (c – a) = 0, only if
(A) a + b + c = 0 (B) a = b = c
(C) a = b or b = c or c = a (D) a – b – c = 0
Sol.

7. If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$ is
(A) 1 (B) a b c (C) 2 (D) 3
Sol.

8. If x – a is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to
(A) 0 (B) 2 (C) – 2 (D) 1
Sol.

9. The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by $(x - 4)$ leave the same remainder. The value of k is

(A) 2 (B) 1 (C) 0 (D) –1

Sol.

10. If $2x^3 - 5x^2 + x + 2 = (x - 2)(ax^2 - bx - 1)$, then a & b are respectively

(A) 2, 1 (B) 2, – 1 (C) 1, 2 (D) –1, 1/2

Sol.

11. Solution of $|4x + 3| + |3x - 4| = 12$ is

(A) $x = -\frac{7}{3}, \frac{3}{7}$ (B) $x = -\frac{5}{2}, \frac{2}{5}$

(C) $x = -\frac{11}{7}, \frac{13}{7}$ (D) $x = -\frac{3}{7}, \frac{7}{5}$

Sol.

12. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol.

13. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) 1/2 (B) 1 (C) 2 (D) 4

Sol.

14. Greatest integer less than or equal to the number $\log_2 15 \cdot \log_{1/6} 2 \cdot \log_3 1/6$ is

(A) 4 (B) 3 (C) 2 (D) 1

Sol.

15. If $\log_x \log_{18}(\sqrt{2} + \sqrt{8}) = \frac{1}{3}$. Then the value of 1000 x is equal to

(A) 8 (B) 1/8 (C) 1/125 (D) 125

Sol.

16. The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$

(C) $a^2 - a + 1$ (D) $a^2 + a + 1$

Sol.

17. $\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_c a+\log_c b}+\frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(A) abc (B) $\frac{1}{abc}$ (C) 0 (D) 1

Sol.

18. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(A) zero (B) 1 (C) 2 (D) more than 2

Sol.

19. Number of real solution of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is

(A) none (B) exactly 1 (C) exactly 2 (D) 4

Sol.

20. Number of real solution (x) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

(A) exactly four (B) exactly three
(C) exactly two (D) exactly one

Sol.

21. The number $\log_2 7$ is

(A) an integer (B) a rational number
(C) an irrational number (D) a prime number

Sol.

22. Anti logarithm of 0.75 to the base 16 has the value equal to

(A) 4 (B) 6 (C) 8 (D) 12

Sol.

EXERCISE – II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. If x & y are real numbers and $\frac{y}{x}$ = x, then 'y' cannot take the value(s)

(A) –1 (B) 0 (C) 1 (D) 2

Sol.

2. Let N = $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is

(A) a natural number (B) a prime number
(C) a rational number (D) an integer

Sol.

3. The solution set of the system of equations

$\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is

(A) (6, 3) (B) (3, 6) (C) (6, 12) (D) (12, 6)

Sol.

4. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(A) one irrational solution (B) no prime solution
(C) two real solutions (D) one integral solution

Sol.

5. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution (D) complex roots

Sol.

EXERCISE – III SUBJECTIVE QUESTIONS

1. Prove that difference of squares of two distinct odd natural numbers is always a multiple of 8.
Sol.

2. Remove the irrationality in the denominator

(i) $\sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}}$

Sol.

(ii) $\dfrac{1}{1+\sqrt{2}+\sqrt{3}}$

Sol.

3. Resolve the following into factors.
(i) $(x - y)^3 - y^3$
Sol.

(ii) $a^3 - \dfrac{1}{a^3} + 4$

Sol.

(iii) $x^3 - 6x^2 + 11x - 6$
Sol.

(iv) $x^3 - 9x - 10$
Sol.

(v) $a^2 (b - c) + b^2 (c - a) + c^2(a - b)$
Sol.

4. Factorize
(i) $1 + x^4 + x^8$
Sol.

(ii) $x^4 + 4$
Sol.

5. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then find the value of $\dfrac{2a^4b^2 + 3a^2c^2 - 5e^4f}{2b^6 + 3b^2d^2 - 5f^5}$ in terms of a and b.
Sol.

6. What can be said about the number, $a_1, a_2, .., a_n$ if it is known that, $|a_1| + |a_2| + |a_3| + ... + |a_n| = 0$.
Sol.

7. Solve the following linear equations

(i) $| x | + 2 = 3$

Sol.

(ii) $| x | - 2x + 5 = 0$

Sol.

(iii) $x| x | = 4$

Sol.

(iv) $| x |^2 - | x | + 4 = 2 x^2 - 3 | x | + 1$

Sol.

(v) $|x - 3| + 2|x + 1| = 4$

Sol.

8. Calculate $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

Sol.

9. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

Sol.

10. If $\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where a, b, c are different positive real number $\neq 1$), then find the value of a b c.

Sol.

11. If $a = \log_{12} 18$ & $b = \log_{24} 54$, then find the value of $ab + 5(a - b)$.

Sol.

12. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, show that $a^a b^b c^c = 1$.

Sol.

13. Which is greater

(a) $\log_2 3$ or $\log_{1/2} 5$

Sol.

(b) $\log_7 11$ or $\log_8 5$

Sol.

(Q. 14 to Q. 22) Solve for x :

14. $\log_{10}(x^2 - 12x + 36) = 2$

Sol.

15. $\log_4 \log_3 \log_2 x = 0$

Sol.

16. $\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x$

Sol.

17. $2 \log_4 (4 - x) = 4 - \log_2 (-2 - x)$.

Sol.

18. $\log_{10}^2 x + \log_{10} x^2 = \log_{10}^2 2 - 1$

Sol.

19. $x^{\frac{\log x + 5}{3}} = 10^{5 + \log x}$

Sol.

20. $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$

Sol.

21. $\log_4 (\log_2 x) + \log_2 (\log_4 x) = 2$

Sol.

22. $5^x \cdot \sqrt[x]{8^{x-1}} = 500$

Sol.

23. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then find
(a) the number of integers in 6^{15}
Sol.

(b) the number of zeros immediately after the decimal in 3^{-100}
Sol.

24. Solve the equation $\log_{100} |x + y| = 1/2$, $\log_{10}y - \log_{10} |x| = \log_{100} 4$ for x and y.
Sol.

25. Find the values of x satisfying the equation $|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2 \log_x 9$.
Sol.

26. Find all real number x which satisfy the equation
$2 \log_2 \log_2 x + \log_{1/2} \log_2(2\sqrt{2}\ x) = 1$.
Sol.

27. Let A denotes the value of

$$\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right) + \log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$$

when a = 43 and b = 57 and B denotes the value of the expression $(2^{\log_6 18}) \cdot (3^{\log_6 3})$. Find the value of (A . B)
Sol.

28. (a) If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3(1 . 2)$ in terms of x and y
Sol.

(b) If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$.

Sol.

29. (a) If $\log_{10}(x^2 - 12x + 36) = 2$

Sol.

(b) $9^{1 + \log x} - 3^{1 + \log x} - 210 = 0$; where base of log is 3.

Sol.

30. Simplify : **(a)** $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$

Sol.

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

Sol.

31. (a) If $\log_4 \log_3 \log_2 x = 0$

Sol.

(b) If $\log_e \log_5 [\sqrt{2x-2} + 3] = 0$

Sol.

32. (a) Which is smaller ? 2 or $(\log_\pi 2 + \log_2 \pi)$.

Sol.

(b) Prove that $\log_3 5$ and $\log_2 7$ are both irrational.

Sol.

33. Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that $2(\log_a c + \log_b c) = 9\log_{ab} c$
Find the largest possible value of $\log_a b$.

Sol.

34. Find the square of the sum of the roots of the equation.
$\log_3 x . \log_4 x . \log_5 x = \log_3 x . \log_4 x + \log_4 x . \log_5 x . \log_5 x . \log_3 x$

Sol.

35. Find the value of the expression

$$\frac{2}{\log_4(2000)^6}+\frac{3}{\log_5(2000)^6}.$$

Sol.

36. Calculate : $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}$

Sol.

37. Simplify $\frac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}}3}}}{409}.\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}\right).$

Sol.

38. Simplify $5^{\log_{1/5}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}}+\log_{1/2}\frac{1}{10+2\sqrt{21}}$.

Sol.

39. Find 'x' satisfying the equation

$4^{\log_{10}x+1}-6^{\log_{10}x}-2.3^{\log_{10}x^2+2}=0$.

Sol.

40. Given that $\log_2 a=s$, $\log_4 b=s^2$ & $\log_{c^2}(8)=\frac{2}{s^3+1}$.

Write $\log_2\frac{a^2b^5}{c^4}$ as a function of 's' (a, b, c > 0, $c\neq 1$).

Sol.

41. Find the value of $49^{(1-\log_7 2)}+5^{-\log_5 4}$.

Sol.

42. Given that $\log_2 3 = a$, $\log_3 5 = b$, $\log_7 2 = c$, express the logarithm of the number 63 to the base 140 in terms of a , b & c.

Sol.

43. Prove that $a^x - b^y = 0$ where $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$. $a > 0, b > 0$ & $a, b \neq 1$.

Sol.

EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Prove the identity ;

$\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N = \dfrac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$

Sol.

2. (a) Solve for x , $\dfrac{\log_{10}(x-3)}{\log_{10}\left(x^2-21\right)} = \dfrac{1}{2}$

Sol.

(b) $\log(\log x) + \log(\log x^3 - 2) = 0$; where base of log is 10 everywhere.

Sol.

(c) $\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2$

Sol.

(d) $5^{\log x} + 5\, x^{\log 5} = 3\,(a > 0)$; where base of log is a.

Sol.

3. If x, y > 0, $\log_y x + \log_x y = \frac{10}{3}$ and xy = 144, then $\frac{x+y}{2} = \sqrt{N}$ where N is a natural number, find the value of N.

Sol.

4. (a) Given : $\log_{10}34.56 = 1.5386$, find $\log_{10} 3.456$; $\log_{10} 0.3456$ & $\log_{10}0.003456$.

Sol.

(b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.

Sol.

(c) If $\log_{10}2 = 0.3010$ & $\log_{10}3 = 0.4771$, find the value of $\log_{10} (2.25)$.

Sol.

(d) Find the antilogarithm of 0.75, if the base of the logarithm is 2401.

Sol.

5. If $\log_{10} 2 = 0.3010$, $\log_{10}3 = 0.4471$. Find the number of integers in :

(a) 5^{200}

Sol.

(b) 6^{15}

Sol.

6. Solve the system of equations : $\log_a x \log_a (xyz) = 48$

$\log_a y \log_a (xyz) = 12$, a >, $a \neq 1$

$\log_a z \log_a(xyz) = 84$

Sol.

7. Let 'L' denotes the antilog of 0.4 to the base 1024. and 'M' denotes the number of digits in 6^{10}
(Given $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$)
and 'N' denotes the number of positive integers which have the characteristic 2, when base of the logarithm is 6. Find the value of LMN.
Sol.

8. $\log_a(x) = x$ where $a = x^{\log_4 x}$.
Sol.

9. $x^{\log x+4} = 32$, where base of logarithm is 2.
Sol.

10. $\log_{x+1}(x^2 + x - 6)^2 = 4$
Sol.

11. $x + \log_{10}(1 + 2^x) = x \,.\, \log_{10}5 + \log_{10}6$.
Sol.

12. $5^{\log x} - 3^{\log x-1} = 3^{\log x+1} - 5^{\log x-1}$, where the base of logarithm is 10.
Sol.

13. $\dfrac{1 + \log_2(x-4)}{\log_{\sqrt{2}}\left(\sqrt{x+3} - \sqrt{x-3}\right)} = 1$
Sol.

14. $\log_5 120 + (x-3) - 2 \,.\, \log_5(1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$
Sol.

15. $\log 4 + \left(1 + \dfrac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3} + 27\right)$.

Sol.

16. If 'x' and 'y' are real numbers such that,

$2\log(2y - 3x) = \log x + \log y$, find $\frac{x}{y}$.

Sol.

17. The real x and y satisfy $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$, find xy.

Sol.

18. Find the real solutions to the system of equations

$\log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4$

$\log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1$

and $\log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0$

Sol.

19. If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$ then prove that $xyz = xy + yz + zx$.

Sol.

20. Given

$a^2 + b^2 = c^2$ & $a > 0$; $b > 0$; $c > 0$, $c - b \neq 1$, $c + b \neq 1$.

Prove that $\log_{c+b}a + \log_{c-b}a = 2 \cdot \log_{c+b}a \cdot \log_{c-b}a$.

Sol.

21. If $\frac{\log_a N}{\log_c N} = \frac{\log_a N - \log_b N}{\log_b N - \log_c N}$ where $N > 0$ & $N \neq 1$, $a, b, c > 0$ & not equal to 1, then prove that $b^2 = ac$.

Sol.

22. Solve the equation

$$\frac{3}{2}\log_4 (x + 2)^2 + 3 = \log_4(4 - x)^3 + \log_4 (6 + x)^3.$$

Sol.

23. Find the product of the positive roots of the equation $\sqrt{(2008)}(x)^{\log_{2008} x} = x^2$.

Sol.

24. Find x satisfying the equation

$$\log^2\left(1+\frac{4}{x}\right) + \log^2\left(1-\frac{4}{x+4}\right) = 2\log^2\left(\frac{2}{x-1}-1\right).$$

Sol.

25. Solve :

$$\log_3\left(\sqrt{x} + \left|\sqrt{x} - 1\right|\right) = \log_9\left(4\sqrt{x} - 3 + 4\left|\sqrt{x} - 1\right|\right)$$

Sol.

26. Solve for x :

$\log^2(4-x) + \log(4-x) \cdot \log\left(x+\frac{1}{2}\right) - 2\log^2\left(x+\frac{1}{2}\right) = 0.$

Sol.

27. If P is the number of integers whose logarithms to the base 10 have the characteristic p , and Q the number of integers the logarithms of whose reciprocals to the base 10 have the characteristic $-q$, show that $\log_{10} P - \log_{10} Q = p - q + 1$.

Sol.

EXERCISE – V JEE PROBLEMS

1. Solve the equation $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$. **[REE 2000, 5 Marks]**

Sol.

2. Number of solution of $\log_4 (x - 1) = \log_2(x - 3)$ is **[JEE 2001(Scr.)]**

(A) 3 (B) 1 (C) 2 (D) 0

Sol.

3. Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then x_0 is **[JEE 2011, 4]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

Sol.

Answer Ex–I — SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. C **2.** A **3.** D **4.** B **5.** A **6.** B **7.** D **8.** C
9. B **10.** A **11.** C **12.** D **13.** B **14.** C **15.** D **16.** D
17. D **18.** B **19.** C **20.** B **21.** C **22.** C

Answer Ex–II — MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. AB **2.** ABCD **3.** AB **4.** ABCD **5.** ABCD

Answer Ex–III — SUBJECTIVE QUESTIONS

2. **(i)** $\sqrt{2}-1$ **(ii)** $\frac{2+\sqrt{2}-\sqrt{6}}{4}$

3. **(i)** $(x-2y)(x^2+y^2-xy)$ **(ii)** $\left(a-\frac{1}{a}+1\right)\left(a^2+\frac{1}{a^2}-a+\frac{1}{a}+2\right)$ **(iii)** $(x-1)(x-2)(x-3)$

(iv) $(x+2)(x^2-2x-5)$ **(v)** $-(a-b)(b-c)(c-a)$

4. **(i)** $(x^4-x^2+1)(x^2+x+1)(x^2-x+1)$ **(ii)** $(x^2-2x+2)(x^2+2x+2)$ **5.** $\frac{a^4}{b^4}$

6. $a_1 = a_2 = a_3 = \ldots\ldots\ldots\ldots = a_n = 0$

7. **(i)** $x = \pm 1$ **(ii)** $x = 5$ **(iii)** $x = 2$ **(iv)** $x = -3, 3$ **(v)** $x = -1$ **8.** 0 **9.** $x = 10$

10. $a\,b\,c = 1$ **11.** 1 **13.** **(a)** $\log_2 3$ **(b)** $\log_7 11$ **14.** $x = 16$ or $x = -4$

15. 8 **16.** {1/3} **17.** {–4} **18.** $\frac{1}{20}, \frac{1}{5}$ **19.** $\{10^{-5}, 10^3\}$

20. $\left\{1, 5, \frac{1}{25}\right\}$ **21.** $x = 16$ **22.** $x = 3$ **23.** **(a)** 12 **(b)** 47

24. $x = 10/3$, $y = 20/3$ & $x = -10$, $y = 20$ **25.** $x = 2$ or 81 **26.** $x = 8$

27. 12 **28.** **(a)** $\frac{xy+2}{2y}, \frac{xy+2y-2}{2y}$ **(b)** 625

29. **(a)** $x = 16$ or $x = -4$ **(b)** $x = 5$ **30.** **(a)** –1, **(b)** $\log_b N$ **31.** **(a)** 8, **(b)** $x = 3$

32. **(a)** 2 **33.** 2 **34.** 3721 **35.** $\frac{1}{6}$ **36.** 9 **37.** 1

38. 6 **39.** $x = \frac{1}{100}$ **40.** $2s + 10s^2 - 3(s^3 + 1)$ **41.** $\frac{25}{2}$ **42.** $\frac{1+2ac}{2c+abc+1}$

Answer Ex–IV | ADVANCED SUBJECTIVE QUESTIONS

2. **(a)** x = 5 **(b)** x = 10 **(c)** $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$ **(d)** $x = 2^{-\log a}$ where base of log is 5. **3.** 507

4. **(a)** 0.5386l $\bar{1}.5386$; $\bar{3}.5386$ **(b)** 2058 **(c)** 0.3522 **(d)** 343 **5.** **(a)** 140, **(b)** 12

6. (a^4, a, a^7) or $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$ **7.** 23040 **8.** x = 2 **9.** x = 2 or $\frac{1}{32}$

10. x = 1 **11.** x = 1 **12.** x = 100 **13.** x = 5 **14.** x = 1 **15.** $x \in \phi$

16. 4/9 **17.** $xy = 2^9$ **18.** x = 1, y = 5, z = 1 or x = 100, y = 20, z = 100

22. x = 2 or $1 - \sqrt{33}$ **23.** $(2008)^2$ **24.** $x = \sqrt{2}$ or $\sqrt{6}$ **25.** $[0, 1] \cup \{4\}$

26. $\left\{0, \frac{7}{4}, \frac{3+\sqrt{24}}{2}\right\}$ **27.** p – q + 1

Answer Ex–V | JEE PROBLEMS

1. x = 3 or – 3 **2.** B **3.** C